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## Top Quark Production Cross Section

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### Abstract

The production rate for top quarks at the Fermilab Tevatron is presented using the exact order  $\alpha_s^3$  corrected cross section and the resummation of the leading soft gluon corrections in all orders of perturbation theory.

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# 1 Introduction

At the Tevatron, the top quark will be mainly produced through  $t\bar{t}$  pair creation. Both top quarks will then decay to  $(W, b)$  pairs, and then each  $W$  can decay either hadronically or leptonically. It is in the channel where both  $W$ 's decay leptonically, one to a  $(e, \nu_e)$  pair, the other to a  $(\mu, \nu_\mu)$  pair, that a large part of the current search effort to find the top quark is concentrated. This is because the background in this channel from  $W^+W^-$  plus jets is fairly small. Using the known branching fractions of the above decays, and taking acceptances into account, an experimental top production cross section is determined, which is then compared with a curve of the theoretical cross section as a function of the top quark mass  $m$ . At present the top quark has not yet been discovered at the Tevatron, and thus an estimate of the theoretical cross section is needed to either determine its mass or establish a lower limit on it. We therefore discuss the estimates of this cross section here.

There have been previous predictions of the top quark cross section based on the results of the fixed order  $\alpha_s^2$  plus  $\alpha_s^3$  contributions in perturbative QCD (pQCD) [1],[2], [3],[4]. As with all fixed order pQCD calculations, these contain a scale (factorization scale = renormalization scale) which reflects the size of the uncalculated  $O(\alpha_s^4)$  and higher order terms. Although the dependence on this scale is relatively flat, indicating that the present result is stable under scale changes, the size of the  $O(\alpha_s^3)$  term is disturbing. In fig.1 we show the ratio  $R = (\sigma_{\text{exact}}^{(0)} + \sigma_{\text{exact}}^{(1)})/\sigma_{\text{exact}}^{(0)}$  for top quark masses in the range relevant to the Fermilab Tevatron. With  $\sigma_{\text{exact}}^{(n)}$  we denote the exact order  $\alpha_s^{(n+2)}$  contribution to the cross section, which implies that  $\sigma_{\text{exact}}^{(0)}$  stands for the Born contribution. Here, and throughout this paper, we have used recent parton distribution functions (MRSD<sub>-</sub>)<sup>2</sup> [5], choosing the DIS factorization scheme, and the two-loop running coupling constant (in the  $\overline{\text{MS}}$  scheme) with five active flavours and  $\Lambda = 0.152$  GeV. The ratio  $R$  of the next-to-leading order (NLO) to the leading-order (LO) term is usually referred to as the K-factor.

One notes from fig.1 that the higher order corrections in the  $q\bar{q}$  channel are small, whereas those in the  $gg$  channel are 70% or larger. Throughout the

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mass range we consider the  $qg$  and  $\bar{q}g$  channel give negligible contributions so we do not consider them. To show which channel has the largest cross section in this mass range we plot the ratios of the NLO  $gg$  and  $q\bar{q}$  contributions to the total result in fig.2. Here we see that as the top quark mass increases the  $q\bar{q}$  channel contribution is larger than the  $gg$  channel one, so the effect of the large K-factor in the latter channel, as seen in fig.1, decreases. However, at masses around  $150 \text{ GeV}/c^2$  the  $gg$  channel still contributes around 20%. This is enough to worry about even higher order corrections.

These large corrections are predominantly from the threshold region for heavy quark production, where we have shown previously that initial state gluon bremsstrahlung (ISGB) is responsible for the large corrections at NLO [6]. To demonstrate this we have run our cross sections programs with a cut on the variable  $\eta = (\hat{s} - 4m^2)/4m^2$ , where  $\hat{s}$  is the square of the parton-parton cms energy so that:

$$\sigma(\eta_{cut}) = \sum_{i,j} \int_0^{\eta_{cut}} d\eta \frac{1}{1+\eta} \Phi_{ij}(\eta, \mu^2) \hat{\sigma}_{ij}(\eta, m^2, \mu^2), \quad (1)$$

where  $\Phi_{ij}$  denotes the parton flux and  $\hat{\sigma}_{ij}$  the partonic cross section for the incoming partons  $i$  and  $j$ . The precise definitions of these functions are given in Ref.[6]. The factorization scale is  $\mu$ , which we have set equal to the renormalization scale. Notice that the maximum value of  $\eta_{cut}$  is given by  $\eta_{max} = (S - 4m^2)/4m^2$ , where  $S$  denotes the square of the total hadronic energy in the cms system. As we increase  $\eta_{cut}$  from small values ( $\simeq 10^{-1}$ ) to larger values, where the actual value of the cross section is approached, there is a rapid rise in  $\sigma(\eta_{cut})$ . Figures 3 and 4 show  $\sigma(\eta_{cut})$  for the  $gg$  and the  $q\bar{q}$  contributions to the cross section. The fact that both cross sections rise sharply around  $\eta_{cut} = 1$  (where  $\hat{s} \approx 8m^2$ ) indicates that the threshold region is very important. This is especially true in the  $gg$  channel, which dominates the total cross section at smaller top quark masses. Both figures also show that the cross section flattens out if  $\eta_{cut}$  is increased further, indicating that partonic processes with  $\hat{s} >> 4m^2$  contribute very little to the cross section.

In a previous paper [7] we carefully examined the dominant logarithms from ISGB which are the cause of the large corrections near threshold. Such logarithms have been studied previously in Drell-Yan (DY) [8] production at fixed target energies (again near threshold) where they are responsible for correspondingly large corrections. In [7] we exploited the analogy between DY and heavy quark production cross sections and proposed a formula to

resum the leading and next-to-leading logarithms in pQCD to all orders. Since the contributions due to these logarithms are positive (when  $\mu = m$ ), the effect of summing the higher order corrections increases the top quark production cross section over that predicted in  $O(\alpha_s^3)$ . This sum, which will be identified with  $\sigma_{\text{res}}$ , depends on a nonperturbative parameter  $\mu_0$ . The reason that a new parameter has to be introduced is due to the fact that the resummation is sensitive to the scale at which pQCD breaks down. As we approach the threshold region other, nonperturbative, physics plays a rôle (higher twist, bound states, etc) indicated by a dramatic increase in  $\alpha_s$ . We chose to simply stop the resummation at a specific scale  $\mu_0$  where  $\Lambda \ll \mu_0 \ll m$  since it is not obvious how to incorporate the nonperturbative physics. Note that our resummed corrections diverge for small  $\mu_0$  but this is *not* physical since they should be joined smoothly onto some nonperturbative prescription and the total cross section will be finite. However, at the moment our total resummed corrections depend on the parameter  $\mu_0$  for which we can only make a rough estimate. See [7] for more details.

Let us begin by showing the effects of including only the leading soft gluon contribution at  $O(\alpha_s^4)$ , which we call  $\sigma_{\text{app}}^{(2)}$ . An explicit expression for this contribution is given in [7]. Here  $\sigma_{\text{app}}^{(2)}$  stands for the approximation to  $\sigma_{\text{exact}}^{(2)}$  where only the leading soft gluon corrections are taken into account. Figure 5 shows three curves for the exact cross section calculated through  $O(\alpha_s^3)$  at the scales  $\mu = 2m, m$  and  $m/2$ . This is the traditional method of estimating the size of uncalculated higher order contributions. For comparison we add to the  $\mu = m$  case the approximate  $O(\alpha_s^4)$  contribution, yielding a total cross section which is not in the range spanned by the previous curves, but slightly above. Therefore, the traditional method does not work very well in this case, due to the size of the corrections, as was already pointed out in [9].

Now we study the effect of the resummation, which depends on  $\mu_0$ , by calculating  $\sigma_{\text{res}}$ . However, because we know the exact  $O(\alpha_s^3)$  result, we can make an even better estimate of the cross section by calculating the quantity

$$\sigma_{\text{imp}} = \sigma_{\text{res}} - \sigma_{\text{app}}^{(1)} + \sigma_{\text{exact}}^{(1)}, \quad (2)$$

which we call the improved cross section. We remind the reader that  $\sigma^{(n)}$  denotes the  $O(\alpha_s^{(n+2)})$  contribution to the cross section. Further  $\sigma_{\text{exact}}^{(n)}$  denotes the exact calculated cross section and  $\sigma_{\text{app}}^{(n)}$  the approximated one where only the leading soft gluon corrections are taken into account. We compare the

results for  $\sigma_{\text{imp}}$  versus the fixed order result  $\sigma_{\text{exact}}^{(0)} + \sigma_{\text{exact}}^{(1)} + \sigma_{\text{app}}^{(2)}$  in fig.6 for various (reasonable) values of  $\mu_0$ . Note that  $\mu_0$  need not be the same in the  $q\bar{q}$  and  $gg$  channels because the convergence properties of perturbation series could be different in these channels and depend on the factorization scheme. For example the cross section due to the  $q\bar{q}$  process seems to converge faster.

As we decrease  $\mu_0$  the cross sections increase. In figure.6 we show three curves for various choices of  $\mu_0$ . The requirement that  $\Lambda \ll \mu_0 \ll m_t$  is satisfied by all three choices. This is not a very restrictive requirement in the sense that it still leaves a large range of values possible, thus rendering the cross section from pQCD more uncertain than thought previously.

In fig.7 we show the contribution from the  $q\bar{q}$  channel to fig.6, while the  $gg$  channel is given in fig.8. The plots show that the range of possible cross sections is quite narrow for the  $q\bar{q}$  channel (due to the smaller relative correction) while it is relatively large in the  $gg$  channel. The curves are calculated in the DIS factorization scheme, where the corrections are smaller because most of them have been absorbed in the parton distribution functions. Thus the resummation is successful for the  $q\bar{q}$  channel, in the sense that  $\sigma_{\text{imp}}$  differs very little from  $\sigma_{\text{exact}}^{(0)} + \sigma_{\text{exact}}^{(1)}$ . This is unfortunately not the case for  $gg$ . We have also checked that these results change very little when using CTEQ [10] parton distribution functions. This is to be expected because, due to the fact that top is so heavy, the cross section is mainly sensitive to parton distribution functions at large  $x$  where they have been well measured.

Finally in Table 1 we present a lower limit estimate (from [11]), and a central value and upper limit estimate. The latter two are also shown in fig.6 as the central and upper solid line respectively, and are obtained as follows. For both estimates we used the improved cross section (2). Each of the three terms in (2) was calculated according to (1), with  $\eta_{\text{cut}} = \eta_{\text{max}}$ . Furthermore, as stated earlier, we used the MRSD<sub>-</sub>' parton distribution functions, and chose the DIS factorization scheme and the two-loop running coupling constant (in the  $\overline{\text{MS}}$  scheme) with five active flavours and  $\Lambda = 0.152$  GeV. The exact partonic cross sections used to calculate  $\sigma_{\text{exact}}^{(0)} + \sigma_{\text{exact}}^{(1)}$  were obtained from [1, 2, 3]. The approximate DIS scheme partonic cross section used for determining  $\sigma_{\text{app}}^{(1)}$  is given explicitly in eqn. (2.10) in [7]. For the partonic resummed cross section we used eqn. (3.24) in the same reference. For all three cross section contributions we chose  $\mu = m$ . However, the central value and upper limit estimates differ by the value chosen for the nonperturbative pa-

rameter  $\mu_0$ . Thus, for the central value we chose  $\mu_0 = 0.1m$  and  $\mu_0 = 0.25m$  for the  $q\bar{q}$  and  $gg$  channels respectively, whereas for the upper limit we chose  $\mu_0 = 0.05m$  and  $\mu_0 = 0.2m$ , respectively. It should be noted that these estimates merely represent what we think are reasonable choices for the parameter  $\mu_0$ , and are thus still not completely rigorous. As far as the lower limit estimate is concerned, note that we can write

$$\sigma_{\text{imp}} = \sigma_{\text{exact}}^{(0)} + \sigma_{\text{exact}}^{(1)} + \sum_{i=2}^{\infty} \sigma_{\text{app}}^{(i)}, \quad (3)$$

Since  $\sigma_{\text{app}}^{(i)} > 0$  for all  $i$  at  $\mu = m$  [7], the true total cross section is likely larger than

$$\sigma_{\text{lower}} = \sigma_{\text{exact}}^{(0)} + \sigma_{\text{exact}}^{(1)} + \sigma_{\text{app}}^{(2)}, \quad (4)$$

so we are justified in using this value as a lower limit. The calculation of the three terms in (4) goes analogously as described above for the central value and upper limit cases. It differs only in the fact that we do not have the nonperturbative parameter  $\mu_0$  here and that we used the conservative value  $\Lambda = 0.105$  GeV in the expression for  $\alpha_s$  when we make this estimate for the lower limit. The explicit expression for  $\sigma_{\text{app}}^{(2)}$  in the DIS scheme is given in eqn. (2.14) in [7].

In conclusion we have demonstrated that the leading and next-to-leading logarithmic corrections to the top quark production cross section are large near threshold and have to be resummed to give a more precise estimate for the cross section. When the ISGB contributions are resummed to all orders in pQCD a new scale  $\mu_0$  has to be introduced, which measures the sensitivity of the cross section to nonperturbative physics. The top quark production cross section at the Fermilab Tevatron is sensitive to this new scale, mainly via the contribution from the gluon-gluon fusion channel.

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$m_{\text{top}}$	$\sigma$ (pb) Lower	$\sigma$ (pb) Central	$\sigma$ (pb) Upper	$m_{\text{top}}$	$\sigma$ (pb) Lower	$\sigma$ (pb) Central	$\sigma$ (pb) Upper
90	148	180	259	146	12.1	13.6	16.2
92	132	160	227	148	11.3	12.6	15.0
94	118	143	204	150	10.5	11.7	13.8
96	106	127	180	152	9.79	10.9	12.8
98	95.2	114	158	154	9.14	10.1	11.9
100	86.3	102	141	156	8.52	9.40	11.0
102	77.8	92.4	127	158	7.94	8.77	10.3
104	70.6	83.2	113	160	7.41	8.16	9.53
106	64.0	75.4	102	162	6.92	7.62	8.82
108	58.1	68.0	90.9	164	6.48	7.11	8.25
110	52.7	61.6	81.4	166	6.07	6.67	7.70
112	48.2	55.9	73.6	168	5.68	6.23	7.18
114	43.9	51.2	66.6	170	5.32	5.83	6.68
116	40.2	46.6	60.6	172	4.98	5.45	6.25
118	36.8	42.4	54.7	174	4.67	5.10	5.83
120	33.7	38.9	49.7	176	4.38	4.79	5.46
122	31.1	35.6	45.4	178	4.11	4.49	5.09
124	28.4	32.6	41.1	180	3.86	4.21	4.78
126	26.2	29.9	37.5	182	3.63	3.94	4.47
128	24.2	27.5	34.5	184	3.40	3.70	4.16
130	22.3	25.4	31.6	186	3.20	3.48	3.92
132	20.6	23.3	29.0	188	3.00	3.27	3.67
134	19.1	21.5	26.5	190	2.83	3.06	3.44
136	17.6	19.9	24.3	192	2.67	2.88	3.22
138	16.3	18.3	22.4	194	2.50	2.70	3.02
140	15.1	16.9	20.5	196	2.36	2.55	2.85
142	14.0	15.7	19.0	198	2.22	2.40	2.68
144	13.0	14.5	17.4	200	2.09	2.26	2.52

Table 1. The first and fifth column contain the top quark mass in  $\text{GeV}/c^2$ . The columns denoted by ‘Lower’ show our lower limit estimate of the top quark cross section in picobarns, the columns denoted by ‘Central’ show our central value estimate, and the columns denoted by ‘Upper’ show our upper limit estimate.

## References

- [1] P. Nason, S. Dawson and R.K. Ellis, Nucl. Phys. B303 (1988) 607; B327 (1989) 49; E.B335 (1990) 260.
- [2] G. Altarelli, M. Diemoz, G. Martinelli, and P. Nason, Nucl. Phys. B308 (1988) 724.
- [3] W. Beenakker, H. Kuijf, W.L. van Neerven and J. Smith, Phys. Rev. D40 (1989) 54; W. Beenakker, W.L. van Neerven, R. Meng, G.A. Schuler and J. Smith, Nucl.Phys. B351 (1991) 507.
- [4] R.K. Ellis, Phys. Lett. B259 (1991) 492.
- [5] A.D. Martin, R.G. Roberts, W.J. Stirling, Phys. Lett. B306 (1993) 145.
- [6] R. Meng, G.A. Schuler, J. Smith and W.L. van Neerven, Nucl. Phys. B339 (1990) 325.
- [7] E. Laenen, J. Smith and W.L. van Neerven, Nucl. Phys. B369 (1992) 543.
- [8] G. Sterman, Nucl. Phys. B281 (1987) 310; D. Appel, P. Mackenzie and G. Sterman, Nucl. Phys. B309 (1988) 259; S. Catani and L. Trentadue, Nucl. Phys. B327 (1989) 323, B353 (1991) 183; H. Contopanagos and G. Sterman, Nucl. Phys. B400 (1993) 211.
- [9] F.A. Berends, J.B. Tausk and W.T. Giele, Phys. Rev. D47 (1993) 2746.
- [10] J. Botts, J.G. Morfín, J.F. Owens, J. Qiu, W.-K. Tung, H. Weerts, Phys. Lett. B304 (1993) 159.
- [11] E. Laenen, preprint FERMILAB-Pub-93/155-T, June 1993.

## Figure Captions

**Fig. 1.** The ratio  $R$  ('K-factor') for the NLO exact top quark cross section as a function of the top quark mass. Plotted are the K-factors for the total cross section (solid line), and individually for the  $q\bar{q}$  channel (long-dashed line) and  $gg$  channel (short-dashed line).

**Fig. 2.** Fraction of  $q\bar{q}$  channel (long-dashed line) and  $gg$  channel (short-dashed line) contribution to total NLO cross section as function of the top quark mass.

**Fig.3** Cross section as function of  $\eta_{cut}$  (see eq.(1)) for  $q\bar{q}$  channel. We used the DIS scheme MRSD<sub>-</sub>' parton distribution functions, and  $m = 100$  GeV/ $c^2$ . Plotted are  $\sigma(\alpha_s^2)$  (solid line) and  $\sigma(\alpha_s^3)$  (dashed line).

**Fig.4** Same as fig.3 but now for the  $gg$  channel.

**Fig.5** The NLO exact cross section as a function of the top quark mass for three choices of scale:  $\mu = m/2$  (upper solid line),  $\mu = m$  (central solid line) and  $\mu = 2m$  (lower solid line), and the NLO exact cross section plus the  $O(\alpha_s^4)$  contribution at  $\mu = m$ , (dashed line).

**Fig.6** The  $O(\alpha_s^4)$  cross section at  $\mu = m$  (dashed line) and  $\sigma_{imp}$  (eq. (2)) for three choices of scale  $\mu_0$ , the two numbers per line corresponding to the  $q\bar{q}$  and  $gg$  channels respectively:  $0.05 m/0.2 m$  (upper solid line),  $0.1 m/0.25 m$  (central solid line),  $0.2 m/0.3 m$  (lower solid line).

**Fig.7** Range of cross sections for  $q\bar{q}$  channel only. The two solid lines span the exact NLO cross section for the range  $m/2 < \mu < 2m$ , and the two dashed lines  $\sigma_{imp}$  for the range  $0.05m < \mu_0 < 0.2m$ .

**Fig.8** Range of cross sections for  $gg$  channel only. The two solid lines span the exact NLO cross section for the range  $m/2 < \mu < 2m$ , and the two dashed lines  $\sigma_{imp}$  for the range  $0.2m < \mu_0 < 0.3m$ .